Jméno	Kombinace

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(1) Určete množinu všech řešení rovnice $\sin^2 2x = \sin^2 x + \frac{1}{2}$ na intervalu $\langle 0; 2\pi \rangle$.

dosadíme ~~vzorec~~ vzorce $\sin 2x = 2\sin x \cos x$

$\boxed{S: \sin^2 x = y}$

$(2\sin x \cos x)^2 = \sin^2 x + \frac{1}{2}$

$4\sin^2 x\,(1-\sin^2 x) = \sin^2 x + \frac{1}{2}$

$4\sin^2 x - 4\sin^4 x = \sin^2 x + \frac{1}{2}$

$-4\sin^4 x + 3\sin^2 x - \frac{1}{2} = 0$

~~$\sin^2 x$~~

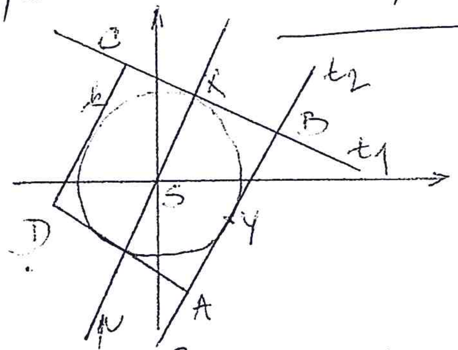
$-4y + 3y - \frac{1}{2} = 0$

$y_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4(-4)(-\frac{1}{2})}}{-4\cdot(2)} = \frac{-3 \pm \sqrt{9-8}}{-8} = \frac{-3 \pm 1}{-8} = \Big\langle \begin{matrix} \frac{1}{2} \\ \frac{1}{4} \end{matrix}$

$\sin^2 x = \frac{1}{2} \Rightarrow |\sin x| = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \underline{x = \frac{\pi}{4} + k\pi}$

$\sin^2 x = \frac{1}{4} \Rightarrow |\sin x| = \frac{1}{2} \Rightarrow \underline{x = \frac{\pi}{6} + k\pi}$ Řešení rovnice je $\{\frac{\pi}{4}; \frac{\pi}{6}\}$.

(2) Kružnici $k = (S; 2{,}5)$; $S[0;0]$; je opsán čtverec ABCD tak, že strana AB je rovnoběžná s přímkou $p = \frac{4}{3}x$. Najděte souřadnice jeho vrcholů.



$x^2 + y^2 = \left(\frac{5}{2}\right)^2$; $x^2 + \frac{16}{9}x^2 = \frac{25}{9}x^2 = \frac{25}{4}$;

$x^2 = \frac{9}{4}$; $x = \frac{3}{2}$; $y = \frac{4}{3} \cdot \frac{3}{2} = 2$

$X = [\frac{3}{2}; 2]$

$t_1 = xx_0 + yy_0 = r^2$

$\frac{3}{2}x + 2y = \frac{25}{4} \Rightarrow \underline{t_1: 6x + 8y = 25}$

$r: y = -\frac{3}{4}x$; $x^2 + \left(-\frac{3}{4}x\right)^2 = x^2 + \frac{9}{16}x^2 = \frac{25}{4}$

$\frac{25}{16}x^2 = \frac{25}{4}$; $x^2 = 4$; $x = 2$

$y = -\frac{3}{4} \cdot 2 = -\frac{3}{2}$ $Y = [2; -\frac{3}{2}]$

$\underline{t_2 = 2x - \frac{3}{2}y = \frac{25}{4}}$

$\left.\begin{matrix} 6x + 8y = 25 \\ 2x - \frac{3}{2}y = \frac{25}{4} \ /\cdot(-3) \end{matrix}\right\} \oplus$

$8y + \frac{9}{2}y = 25 - \frac{75}{4}$ /·4

$32y + 18y = 100 - 75$

$50y = 25$

$y = \frac{1}{2}$

$x = \frac{11}{2}$

$S = \frac{B+D}{2}$

$\underline{B = [\frac{11}{2}; \frac{1}{2}]}$

$\underline{D = [-\frac{11}{2}; -\frac{1}{2}]}$

Jméno	Kombinace

Josef Brejr

① Urcčte množinu všech řešení rovnice $\cos^2 2x = \cos^2 x - \frac{1}{2}$ na intervalu $\langle 0, 2\pi \rangle$.

$\cos 2x = \cos^2 x - \sin^2 x$, ~~$\sin 2x = 2\sin x \cos x$~~, $\sin^2 x + \cos^2 x = 1$

$$(\cos^2 x - \sin^2 x)^2 = \cos^2 x - \frac{1}{2}$$

$$(\cos^2 x - (1 - \cos^2 x))^2 = \cos^2 x - \frac{1}{2}$$

$$(2\cos^2 - 1)^2 = \frac{1}{2}(2\cos^2 x - 1)$$

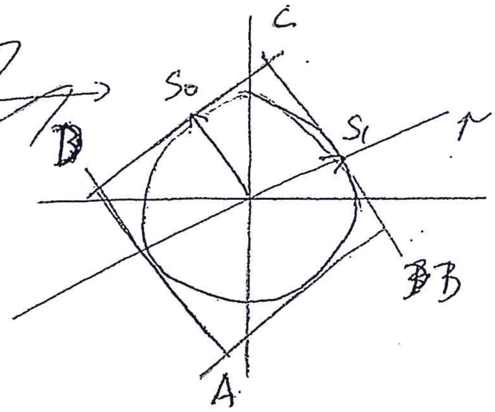
$$2\cos^2 x - 1 = \frac{1}{2}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \frac{\sqrt{3}}{2} \qquad \cos x = \frac{-\sqrt{3}}{2}$$

$$x = \frac{\pi}{6} = 60° \qquad x = \frac{-\pi}{6} = -60°$$

② Kružnici $k = (S; 2{,}5)$; $S[0;0]$ je opsán čtverec ABCD tak, že strana AB je rovnoběžná s přímkou $p: y = \frac{3}{4}x$. Najděte souřadnice jeho vrcholů.



$$\vec{A_p} = \left(1, \frac{3}{4}\right); \quad |A_p| = \sqrt{1 + \left(\frac{3}{4}\right)^2} = \sqrt{\frac{16}{16} + \frac{9}{16}} = \frac{5}{4}$$

$$\vec{SS_1} = 2{,}5 = \frac{5}{2}$$

$$SS_1 = \frac{\left(1, \frac{3}{4}\right)}{\frac{5}{4} \cdot 2} \cdot \frac{5}{2} = \left(\frac{1}{2}; \frac{3}{8}\right)$$

$$SS_0 = \left(-\frac{3}{8}; \frac{1}{2}\right)$$

$$\vec{SC} = \vec{SS_0} + \vec{SS_1} = \left(\frac{1}{8} - \frac{3}{8}; \frac{3}{8} + \frac{1}{2}\right) = \left(\frac{1}{4}; \frac{7}{8}\right)$$

$$C = \left[\frac{1}{4}; \frac{7}{8}\right]$$

$$\frac{A + C}{2} = S$$

$$A = \left[-\frac{1}{4}, -\frac{7}{8}\right]$$

$$\vec{SB} = \vec{SS_1} - \vec{SS_0} = \left(\frac{7}{8}, \frac{1}{8}\right)$$

$$B = \left[\frac{7}{8}, \frac{1}{8}\right]$$

$$D = \left[-\frac{7}{8}, -\frac{1}{8}\right]$$